Name: ______________________________

Statistical Inference Worksheet

MA 151, Spring 2021

Write your solutions to the following problems on clean sheets of notebook or printer paper. Carefully organize your solutions, and label the parts ((a), (b), (c), etc.).

Show all work and explain your reasoning. Use R where appropriate.

Perform all hypothesis tests and construct all confidence intervals directly, and check your answers using `one.sample.t.test` from the `MUsaic` package.

When you complete a problem, raise your hand so I can review your solution. If I am currently working with another student, move on to the next problem, then get my attention when I am done working with that student.

Submit your solutions as a **collated PDF** to eCampus before the beginning of Lecture 17. There is also a Sapling assignment due at the beginning of Lecture 17.

1. According to the norms established for a reading comprehension test, eighth graders should average 84.3 on a standardized test. If 45 randomly selected eighth graders from a certain school district averaged 87.8, and the standard deviation of their scores was 10.1, test the claim that the students exceeded the standard at the 0.01 significance level.

   (a) State the claim in terms of an equality / inequality involving the parameter(s) of the population(s).

   (b) State the null and alternative hypotheses associated with the claim.

   (c) Determine a relevant test statistic for investigating the claim. What sampling distribution will you assume for the test statistic? What must be true about the sample for this sampling distribution to be correct?

   (d) Compute the observed test statistic from the sample statistics.

   (e) Test the claim using a rejection region. Sketch the sampling distribution of the test statistic, and show the rejection region and the observed value of the test statistic in your sketch.

   (f) Test the claim using a P-value. Sketch the sampling distribution of the test statistic, and the area used to computed the P-value.

   (g) State your conclusion as it relates to the original claim.

   (h) Compute a 99% confidence interval for the population parameter.

2. A company that manufacturers coffee for use in commercial machines monitors the caffeine content in its coffee. The company selects 50 samples of coffee every hour from its production line and determines the caffeine content. During a 1-hour time period, the 50 samples yielded a mean caffeine content of 110 mg and the standard deviation across the caffeine contents was 7.1 mg. Test the claim that the mean caffeine content across all coffee manufactured by the compnay is at most 100 mg at the 0.025 significance level.

   (a) State the claim in terms of an equality / inequality involving the parameter(s) of the population(s).

   (b) State the null and alternative hypotheses associated with the claim.

   (c) Determine a relevant test statistic for investigating the claim. What sampling distribution will you assume for the test statistic? What must be true about the sample for this sampling distribution to be correct?

   (d) Compute the observed test statistic from the sample statistics.

   (e) Test the claim using a rejection region. Sketch the sampling distribution of the test statistic, and show the rejection region and the observed value of the test statistic in your sketch.

   (f) Test the claim using a P-value. Sketch the sampling distribution of the test statistic, and the area used to computed the P-value.

   (g) State your conclusion as it relates to the original claim.

   (h) Compute a 97.5% confidence interval for the population parameter.

3. A program to reduce recidivisim has been in effect for two years in a large northeastern state. A sociologist investigates the effectiveness of the program by taking a random sample of 200 prison records of repeat offenders. The records were selected from the files in the courthouse of the largest city in the state. The average length of time out of prison between the first and second offenses is 2.8 years with a standard deviation of 1.3 years. Test the claim that the the average length of time out of prison amongst all repeat offenders is less than 2.5 years at the 0.2 significance level.

   (a) State the claim in terms of an equality / inequality involving the parameter(s) of the population(s).

   (b) State the null and alternative hypotheses associated with the claim.

   (c) Determine a relevant test statistic for investigating the claim. What sampling distribution will you assume for the test statistic? What must be true about the sample for this sampling distribution to be correct?

   (d) Compute the observed test statistic from the sample statistics.

   (e) Test the claim using a rejection region. Sketch the sampling distribution of the test statistic, and show the rejection region and the observed value of the test statistic in your sketch.

   (f) Test the claim using a P-value. Sketch the sampling distribution of the test statistic, and the area used to computed the P-value.

   (g) State your conclusion as it relates to the original claim.

   (h) Compute a 80% confidence interval for the population parameter.

4. 90 adult male patients were enrolled in a study following a new treatment for congestive heart failure. One of the variables measured on the patients was the increase in exercise capacity (in minutes) over a 4-week treatment period. The previous treatment regime had produced an average increase of 2 minutes. The researchers wanted to evaluate whether the new treatment had increased the average exercise capacity compared to the previous treatment. Amongst the 90 patients, the average increase in exercise capacity was 2.17 minutes, and the standard deviation in the exercise capacity increases across the patients was 1.05 minutes. Test the claim that the change in the exercise capacity due to the new treatment is the same as for the old treatment at the 0.001 significance level.

   (a) State the claim in terms of an equality / inequality involving the parameter(s) of the population(s).

   (b) State the null and alternative hypotheses associated with the claim.

   (c) Determine a relevant test statistic for investigating the claim. What sampling distribution will you assume for the test statistic? What must be true about the sample for this sampling distribution to be correct?

   (d) Compute the observed test statistic from the sample statistics.

   (e) Test the claim using a rejection region. Sketch the sampling distribution of the test statistic, and show the rejection region and the observed value of the test statistic in your sketch.

   (f) Test the claim using a P-value. Sketch the sampling distribution of the test statistic, and the area used to computed the P-value.

   (g) State your conclusion as it relates to the original claim.

   (h) Compute a 99.9% confidence interval for the population parameter.

5. An airline wants to test the null hypothesis that 60 percent of its passengers object to smoking inside a plane. They poll a random sample of 100 airline passengers, and find that 58 percent of those polled object to smoking inside a plane. Explain under what conditions they would be committing a type I error and under what conditions they would be committing a type II error.