MA 115: Lab 1 Functions in Excel

13 September 2019

Important Details

- **Due:** Beginning of class on Wednesday, September 18.
- Complete the activities listed in the Hands On sections of this worksheet using the Lab1.xlsx document listed on the course website as a template for the lab. Write your answers directly in the boxes of the worksheet.
- Save the Lab1.xlsx as you go. Rename the file <Last Name>-Lab1.xlsx. Upload the completed file to the Lab 1 assignment on eCampus by the beginning of class on Wednesday, September 18.

The goal of today's lab is to familiarize ourselves with using Excel to generate input-output relationships. In mathematics, we call an input-output relationship a **function**. Recall from the first class session when we defined a function as a 'box' that maps an input (say x) to an output (say y). The schematic below is useful to keep in mind when thinking of a function: we put an input into the function, and the function return one (and only one!) output.

$$x \longrightarrow f \longrightarrow y = f(x)$$

Figure 1: A schematic of a function. An input x is fed into the function f, which outputs y = f(x).

In the remainder of this course, we will learn about many different types of functions f: polynomial, exponential, logarithmic, and trigonometric. We will preview polynomial functions in this lab, and explore the others in later labs.

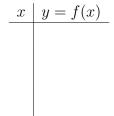
1 Generating Inputs and Outputs of a Function in Excel

The input-output relationship that defines a function is perfect for Excel: we can specify the inputs in one column, and tell Excel to compute the outputs in the other column.

Let's begin by practicing with a simple function. It takes the input x, squares it, and adds 1:

$$y = f(x) = x^2 + 1 (1)$$

In Lecture 3, we talked about how to get a rough idea about the behavior of a function by seeing how it behaves with a small set of inputs. We put a collection of inputs on the left, and the outputs on the right, and create the following table:



We could fill out this table by hand, but computers are much better at doing arithmetic than we are. So let's learn how to use Excel to populate this table with a few test values.

Excel labels rows using numbers and columns using letters. This gives a sort of coordinate system for the spreadsheet. I will refer to the cell in row 1 and column A as **A1**, for example.

Enter some values into column \mathbf{C} , starting at entry $\mathbf{C2}$. Start with the values: -2, -1, 0, 1, 2. You enter values by clicking on the appropriate cell, typing the value, and pressing Enter.

To generate the outputs for these inputs, we need to specify an Excel Formula. Excel Formulas are indicated by beginning the entry of a cell with an equal sign. So to generate an output for the function $y=f(x)=x^2+1$, we click on cell **D2**, and type

$$=C2^2+1$$

This indicates that we are providing a formula, and we have substituted the cell value C2 for the variable x. After pressing enter, you should see that Excel has output 5. You could fill out the remaining cells by-hand, but Excel can also auto-fill a column of a table, entering the appropriate formula in each cell. To do so, click on cell $\mathbf{D2}$, and then double click on the green square at the bottom right-hand corner of the cell.

Fill out the table below using the values reported by Excel. Do these values match what you would have computed by hand?

$$\begin{array}{c|c} x & y = f(x) \\ \hline \end{array}$$

Generating the inputs of the function manually can get tedious. Again, Excel has the functionality to automate the generation of these values. We will next create a range of inputs, starting with -3, ending at 3, with a step size of 0.1. So we will create the inputs $-3, -2.99, -2.98, \ldots, 2.99, 3$.

We want to start at -3. Click on cell **F2** and enter -3. Click on Fill in the right side of the top panel of Excel, and select Series... from the drop-down menu. See **Screenshot** 1 through the **Screenshots for Lab** 1 link on the course website if you are having trouble finding the Fill option. A dialog box will pop up. This dialog box gives us several options. Set Series in to Columns, set Step value: to 0.1, and set Stop value: to 3.5 See **Screenshot** 2 for what this should look like. This will generate the sequence of values from -3 (the initial value in the cell), in increments of 0.1, to 3. Click OK, and Excel will generate the desired input.

Now enter the appropriate formula into cell **G2**, and expand the formula down the column by double-clicking on the green square at the bottom right-hand corner of the cell.

Now we have generated many more input-output values with very little work. This table is one way to think of a function: we list the inputs in one column, and the outputs in the other.

2 Plotting the Graph of a Function Using Excel

Another way to view a function is in terms of its graph: the set of (x,y) points in the coordinate plane that satisfy the equation y=f(x). We have already created the ordered pairs as the columns of our table. We can plot them in the coordinate plane by using Excel's Chart functionality.

Hands On 3

To create a Chart from the table in columns ${\bf F}$ and ${\bf G}$, select those columns by clicking on ${\bf F}$ and then Shift-clicking on ${\bf G}$. This will highlight the two columns. Now choose the Insert tab from the menu bar at the top of the screen, and click on the scatter option. The scatter option is the button with blue and yellow points in the bottom center of the 3×3 grid to the right of the Recommended Charts button. We want to use the Scatter with Straight Lines option, which is the bottom-middle option under Scatter in the drop-down menu. See **Screenshot 3** if you are having trouble finding this. You should now a parabola, the U-shaped curve that is the graph of a quadratic function.

The bare-bones Chart does not look like the graphs of function we are used to from the textbook. We can make a nicer version by clicking on the Chart, and selecting Quick Layout from the menu pane. Select Layout 10, the bottom left layout. See **Screenshot 4** if you are having trouble finding this. Now the Chart looks more like the coordinate plane and graphs we see in our textbook.

We should always label our axes.

To change the labels on the horizontal and vertical axes, double click on the Axis Title, and enter the appropriate label. In this case, label the horizontal axis as x and the vertical axis as y.

We can also query the Chart to learn properties of the graph. For example, we can find the x- and y- intercepts of the graph.

Hands On 5

Hover the mouse pointer over the y-intercept of the graph. What is the coordinate of the y-intercept?

Does the graph have an x-intercept. Explain.

Next, we move on to more complicated functions, and see how to use Excel to generate their tables and graphs.

3 Jill and Jack Go Up a Hill - Polynomial Functions

A topographer has surveyed a new mountain, and found that the height of the mountain above sea level (in feet) h is well-modeled as a function of the distance east (in miles) x by

$$h(x) = -(x - 10)^4 + 2(x - 10) + 10000.$$
 (2)

She has sent an expedition to summit the mountain, and the climbers need to know where exactly to find the peak of the mountain, and how tall the mountain is. In this example, we will help her and her team find the location and height of the peak.

Select the sheet Mountain in the bottom panel of the Excel window. Begin by generating a grid of miles in the x column from 0 to 20 in increments of 0.1. Then populate the h column by entering the appropriate formula.

WARNING: Careful when entering the formula! Excel is less knowledgeable than we are about 2(x-10) being shorthand for $2^*(x-10)$ and $-(x-10)^4$ being shorthand for $-1^*(x-10)^4$. You will need to explicitly enter any arithmetic operation in the formula, and avoid using any shorthand.

Plot the graph of the function h(x) using one of Excel's Chart functionality, as you did in the previous section.

From the initial survey, it is clear that the mountain *has* a peak, but not so clear where that peak is. To aid the expedition, we will need to zoom in closer to the summit.

Hands On 7

We have two option for highlighting the summit: we can create a new table with x values closer to the summit, or we can use Excel's Format Axis option. Let's use the latter. To activate Excel's Format Axis panel, double click on the horizontal axis of the Chart. A panel will open on the right side of the screen. Select the Axis Options icon (which looks like a bar chart) in the top row of the panel (the same row as the paint can). Then select the Axis Options item. See **Screenshot 5** if you are having trouble finding this. You can set the horizontal limits of the plot by changing the Bounds option.

Modify the bounds on both the horizontal and vertical axes until you have isolated the peak of the mountain. **Caution:** the top of the mountain is known to be asymmetric!

How high above sea level is the summit? How far east is the summit? What is the coordinate of the summit? Report all values to one decimal place.

4 There Are Always More Fish in the Sea

We revisit a slightly different model of a fish population. A famous ichthyologist at Monmouth has discovered that the population of fish (in thousands) living in a nearby lake is captured by a

simple 4th degree polynomial,

$$P(t) = 2 + 13.3778t - 6.5939t^2 + 1.0597t^3 - 0.0536t^4,$$
(3)

where t is the number of years since January 1, 2018. He warns that the function predicts a fast rise in the population, followed by a near bust. In order to intervene, he needs to know when exactly the bust will happen. His friend, a fisherman, wants to know when the population will reach Peak Fish.

Hands On 8

Select the sheet Fishes in the bottom row of the Excel window. Begin by generating a grid of years in the t column spanning the 6 years after January 1, 2018, in increments of 0.05 years. Then populate the P column by entering the appropriate formula.

When should the ichthyologist's friend head out to fish? How many fish can she expect?

By what year should the ichthyologist expect the fish population to reach its lowest point after the peak year? How many fish are in the lake at this time?

The ichthyologist returns from her latest field work excited that his intervention seems to have worked: the fish population appears to be on the rise again. She hopes that her model still holds up further into the future.



Extend the range of years for another 4 years. Repopulate the ${\cal P}$ column.

What should the ichthyologist tell his friend?

What happens 10 years after January 1, 2018? Does the model for the fish population still make sense after this time? Why or why not?

5 Reminders

- This completed worksheet and the .xls file you used to complete it are due at the beginning of class on Wednesday, September 18. See the box at the beginning of this worksheet for submission instructions.
- Be sure that you have labeled all axes in any of the Charts that you generated. In the cases where the variables have units, be sure you have included the units.